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THROUGHPUT FOR TIME-CAPTURE SPREAD SPECTRUM

INTRODUCTION

At several well-attended packet radio working group meetings, the use of spread-spectrum signals has been suggested. Among the various advantages which have been considered is the "time-capture" effect possible because the auto-correlation-function of a spread-spectrum signal has a narrow major peak and low side-lobes. The purpose of this note is to propose a model for the time-capture effect and to analyze channel throughput using this model.

In this note we ignore the effects of multipath. This is done to compare spread-spectrum performance with other channel access techniques (such as classic Aloha) under ideal conditions. In fact, this greatly favors spread-spectrum, since multipath will smear the correlation-peak and greatly reduce the effectiveness of time-capture in many situations. The effects of multipath on spread-spectrum may be considered in another note.

Even for this simple situation we find that a spread-spectrum classic-Aloha mode will not perform as well as a slotted Aloha mode. To achieve performance better than slotted Aloha, the spread spectrum system must also be slotted.

Model

We use the model originally suggested by Abramson in the Fall of 1970 AFIPS Conference Proceedings which assumes an infinite population of unperturbable users generating messages and retransmissions at an

aggregate rate λ messages per sec. The message duration is T_m. The problem is to find the rate at which messages can be successfully transmitted using a spread-spectrum modulation format.

The spread-spectrum system which we will consider operates as shown in Figure 1. An encoded binary data-stream is delivered to the modulator whenever a message (packet) is sent. The modulator selects one of two wideband signals depending on whether the message bit is a "one" or a "zero." The wideband signals are "spread-spectrum" in the sense that the bandwidth of each signal is much greater than the minimum bandwidth required to send the information at the basic bit rate. Let the signal rf bandwidth be W and the bit duration be T. Then the signal is spread-spectrum in the sense that $TW \gg 1$. The parameter $N = 2\,TW$ is a measure of the degree of "spread" and $\sqrt{N}$ is commonly called the processing gain of the signal. A bound on the number of orthogonal waveforms of duration T which exist in a band W is $N+1$.

To provide a focus for analysis we have selected a form of spread-spectrum transmission using binary-orthogonal signalling and non-coherent reception. The results we obtain for throughput will be independent of the form of modulator/receiver and depend only on the processing gain, or time-bandwidth product N; however, the analysis is more easily visualized using a specific modulator/receiver.

The receiver consists of a pair of matched filters--one matched to each signal. The filter outputs are envelope-detected and subtracted. The result is sampled synchronously with the received bit-train and a decision is based on the sample.

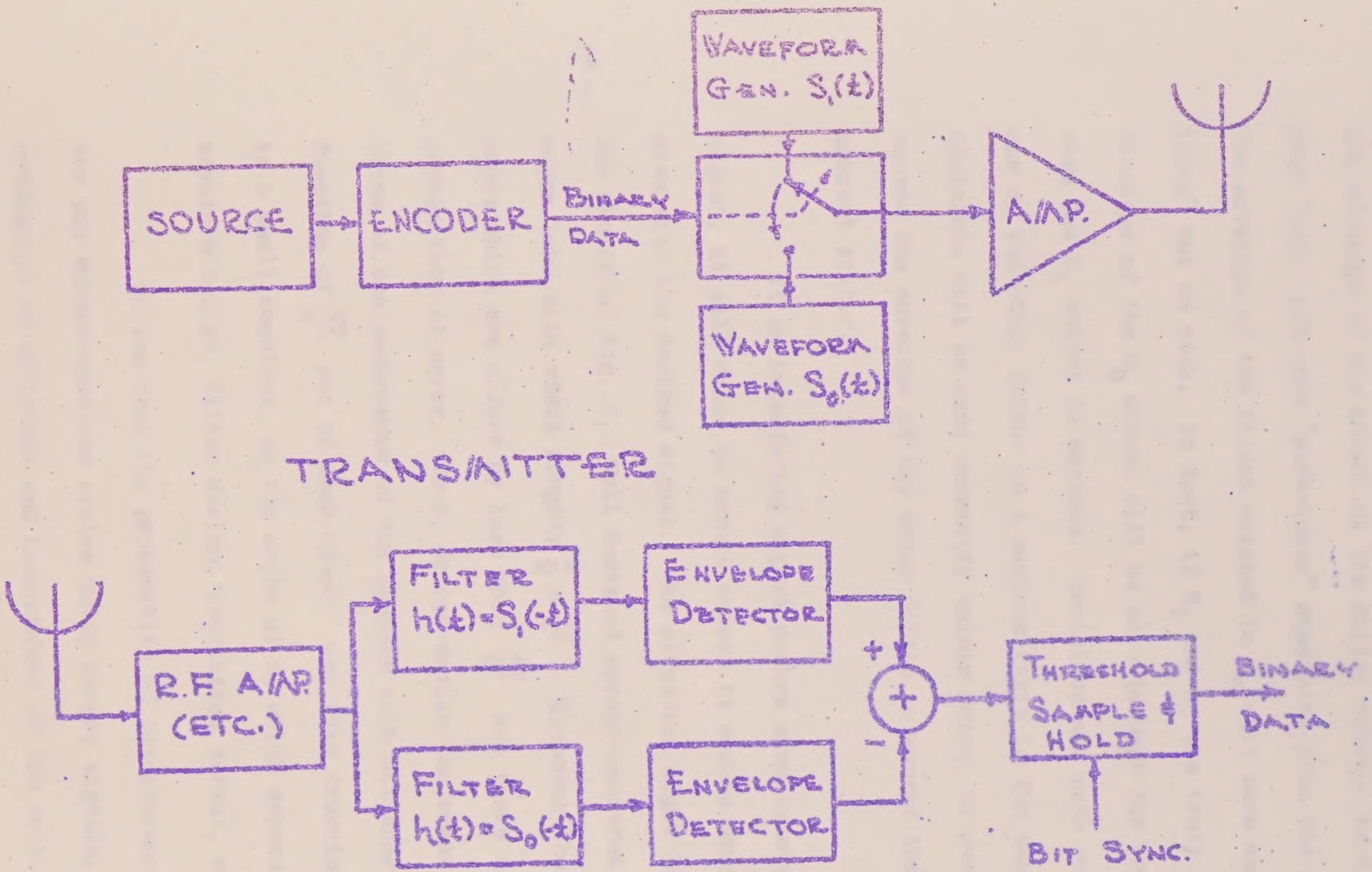
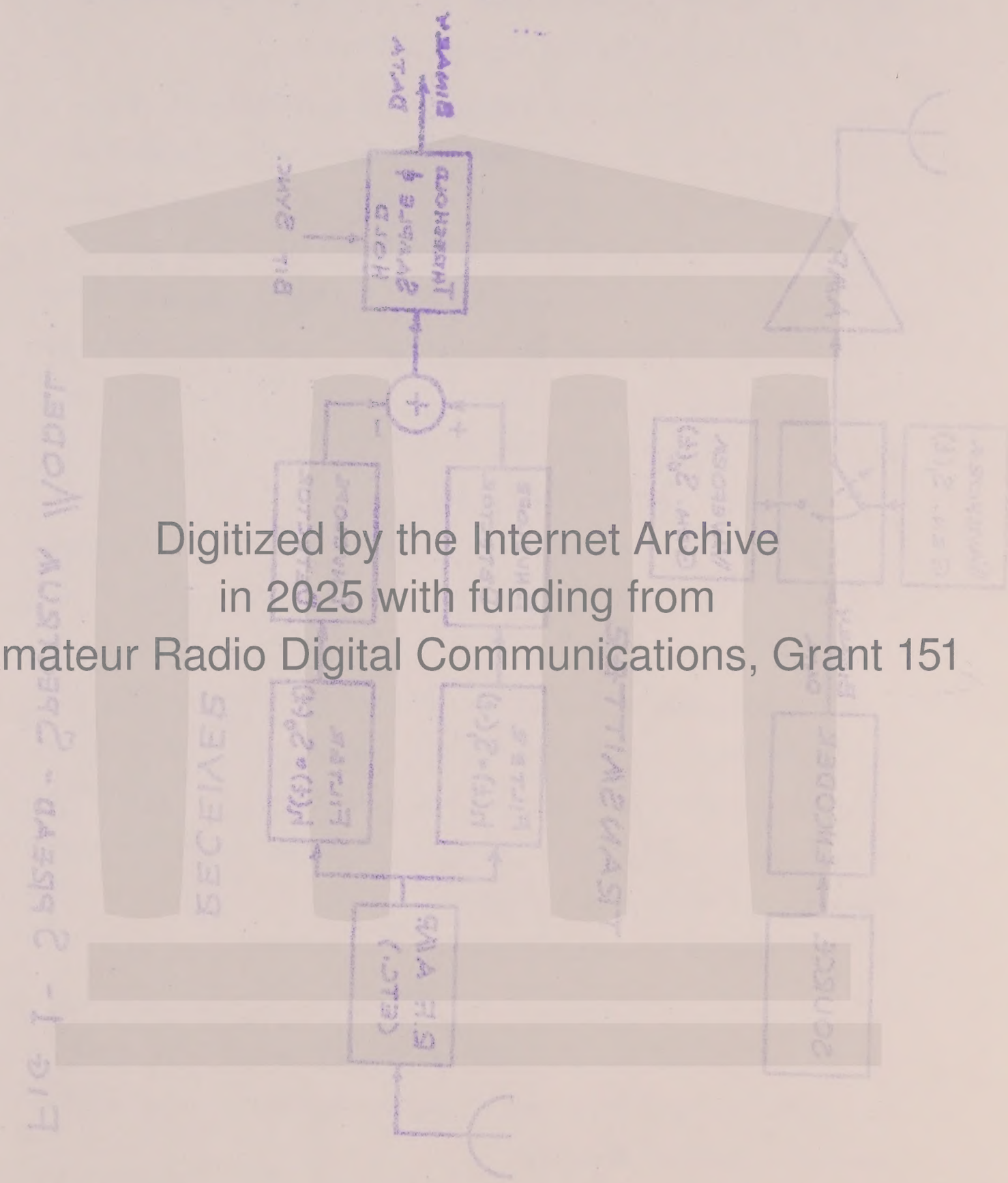


FIG 1 - SPREAD-SPECTRUM MODEL



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Suppose that a single bit, say a "one" is transmitted. Then the envelope of the output of the filter matched to S_1 will have a peak "lobe" and many "side-lobes" something like the sketch of Fig. 2. The envelope of the filter matched to S_0 will have many little "side-lobes" but no peak. In fact, if S_1 and S_0 are truly orthogonal, the envelope of the S_0 output will be zero (except for noise) at the moment when the S_1 output is maximum. Decisions are made only at moments when one or the other filter is a maximum (this is the sampling instant) hence decisions will be made correctly unless noise, or some interfering signal causes the envelope of the wrong filter to exceed the envelope of the correct filter.

If an interfering signal occurs simultaneously with the desired signal, it will cause no errors unless it occurs approximately in synchronism with the desired signal. The situation might look something like the sketch of Fig. 3. Well designed spread-spectrum signals will have a major lobe with width roughly $\frac{2}{W} = \frac{4T}{N}$. Whenever the two signals have major lobes which are offset by less than $\frac{4T}{N}$ sec there will be increased probability of error. Thus, we can define the event "interference" in terms of the occurrence of two signals with bit-start times within some fraction of $\frac{4T}{N}$ sec of each other. Let this fraction by $\frac{k}{2}\ \frac{T}{N}$, where k is a small constant, on the order of 4, which depends on the details of waveform design, filter design, modulation format, etc.

Notice that the probability of interference would be the same for any spread-spectrum system using binary signals, whether they were orthogonal or antipodal and independent of the detection/decision

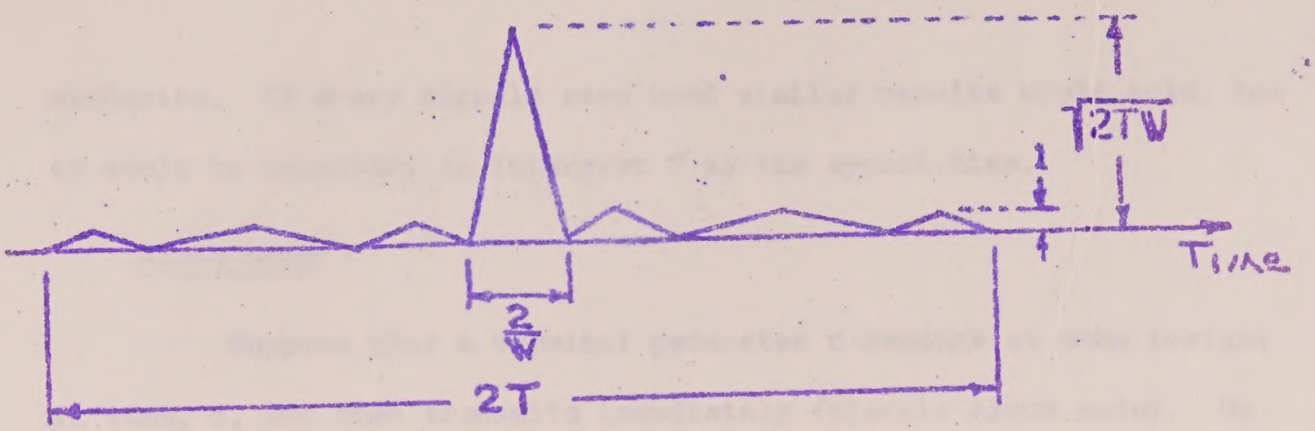


FIG 2- TYPICAL ENVELOPE OF MATCHED FILTER OUTPUT FOR BINARY-PHASE-CODED WAVEFORM.

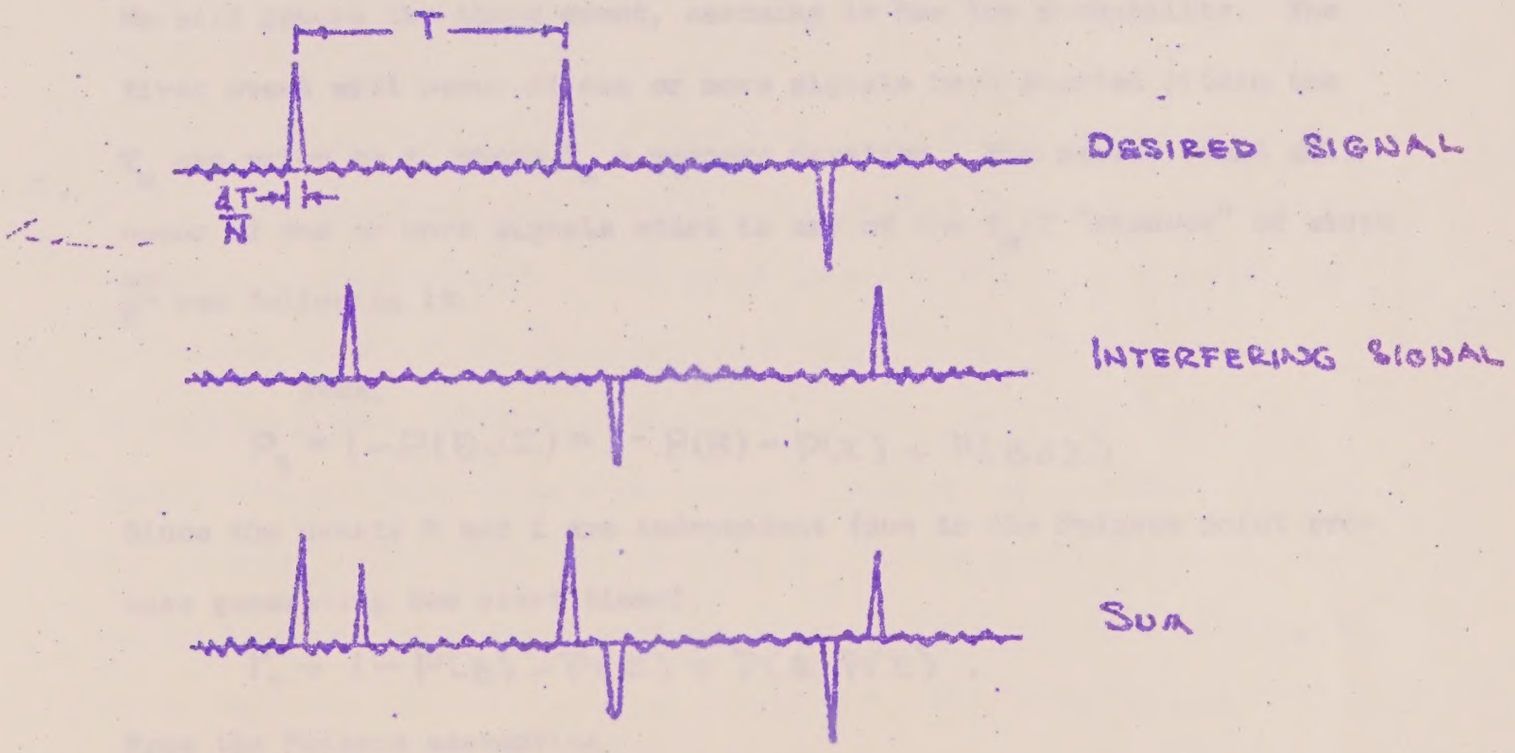
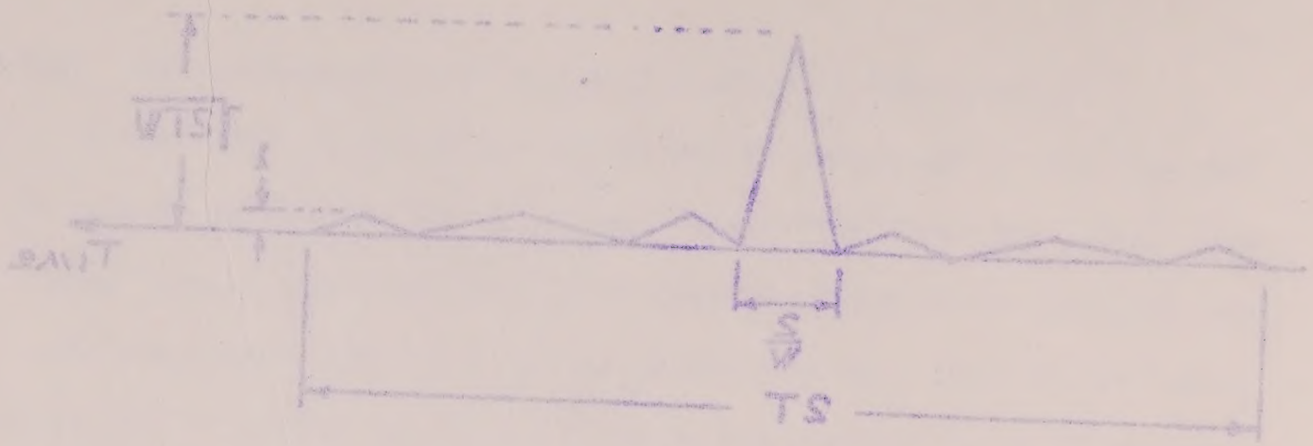


FIG 3.- EFFECT OF INTERFERENCE (NO MULTIPATH)



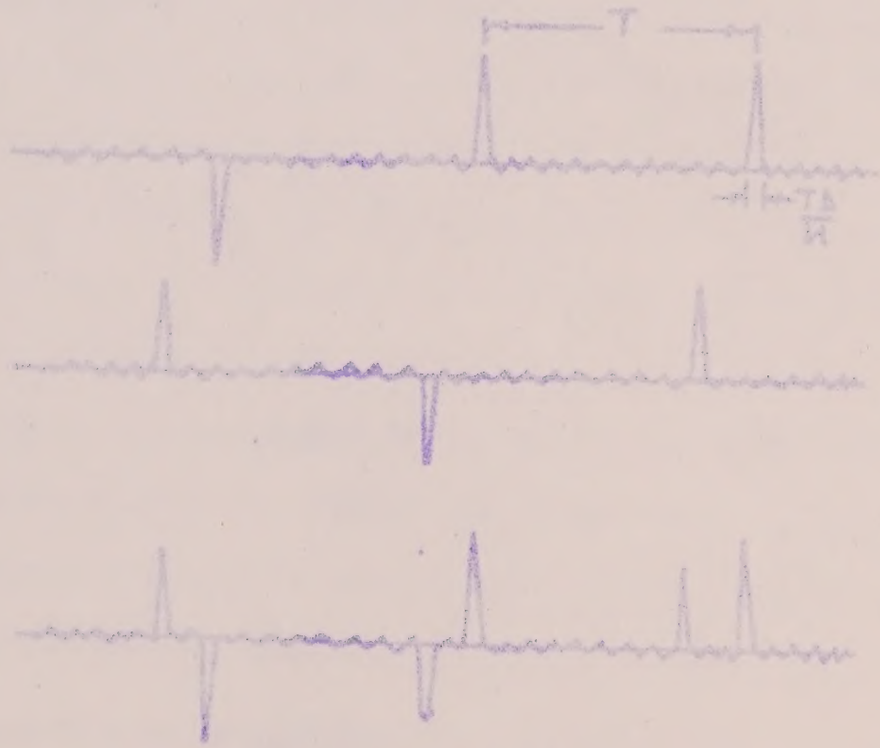


FIG 3 - EFFECT OF INTERFERENCE
(NO MULTIPATH)

mechanism. If M-ary signals were used similar results would hold, but it would be necessary to interpret T as the symbol time.

Aloha Mode

Suppose that a terminal generates a message at some instant in time, t, and then transmits immediately (classic Aloha mode). We would like to calculate the probability that the transmission will be successfully received, P_s.

There are three ways in which a failure may occur

B = the receiver may be busy

I = the signal may be interfered with or

E = noise may cause an error.

We will ignore the third event, assuming it has low probability. The first event will occur if one or more signals have started within the T_m sec prior to t, where T_m = message duration. The second event will occur if one or more signals start in any of the T_m/χ "windows" of width $\frac{kT}{N}$ sec following it.

Thus,

$$P_s = 1 - P(B \cup I) = 1 - P(B) - P(I) + P(B \cap I)$$

Since the events B and I are independent (due to the Poisson point process generating the start times),

$$P_s = 1 - P(B) - P(I) + P(B)P(I) .$$

From the Poisson assumption

$$P(B) = 1 - e^{-\lambda T_m}$$

and

$$P(I) = 1 - e^{-\lambda\left(\frac{kT}{N}\right)\left(\frac{T_m}{T}\right)}$$

so that

$$P_s = e^{-\lambda T_m (1 + \frac{k}{N})}$$

The channel throughput, S, is just

$$S = \lambda T_m P_s$$

so that if we define the channel traffic G as $G = \lambda T_m$,

$$S = G e^{-G(1 + \frac{k}{N})}$$

Notice that if $N \to k$ we have the equation for a classic Aloha channel, and if $N \to \infty$ (very wideband signal) we have the equation for a simple slotted Aloha channel. For $k \leq N \leq \infty$ the channel throughput will lie between Aloha and slotted Aloha.

This result is mildly surprising. One might expect that as $N \to \infty$, perfect time-capture would occur so that the throughput should approach 1 asymptotically. This would indeed be the case if we allowed a separate receiver for each user; however, with only one receiver, the throughput cannot exceed that of slotted Aloha. This is because the channel can be "busy" with either a valid message, or with interference. For example, suppose that the receiver is handling a valid message, and a second terminal generates a message. Then if $N \to \infty$ no interference will occur, but when the valid message ceases, the interfering message will capture the receiver and hold it until the interfering message is complete. The receiver will be "busy" handling traffic which does not contribute to throughput, hence throughput will not approach one.

Notice that throughput can be increased in several ways. First, the busy probability could be reduced by providing multiple samplers and decision circuits. But this increase in performance is bought essentially by using multiple time-slot receivers. It can be matched in an Aloha system by using multiple frequency-slot receivers. Furthermore, the Aloha system will require much less bandwidth.

Slotted Spread-Spectrum

The busy probability can also be reduced by operating in a "slotted" mode. In this case, messages can only start during short periods at the beginning of each slot. The uncertainty in slot-time is necessary because different terminals are at different distances from the relay, and will synchronize to the slots at different times, unless the propagation times are known to the terminals.

A terminal at range r will be blocked by any terminal at shorter range that starts during the same slot. He will be interfered with by terminals at certain longer ranges to be calculated. To approximately calculate throughput, assume that all terminals generate and retransmit traffic with the same message rate (see AES Note 8).

Then the fraction of terminals which might block a terminal at r is $\frac{r^2}{R^2}$ where R is the maximum range of the terminal. This set of terminals will generate messages at a rate $\frac{r^2}{R^2}\lambda$, so that the probability of the receiver being busy is

$$P(B) = 1 - e^{-\frac{r^2}{R^2}\lambda T_m}$$

$$F(E) = 1 - e^{-\frac{E}{kT}}$$

Terminals at ranges such that the bit start times will overlap within $\pm\frac{T}{N}\frac{k}{2}$ of the terminal at range r will cause interference.

If c = velocity of light, and we ignore multipath problems, there are

$$\frac{\left(r+\frac{cTk}{2N}\right)^2-\left(r-\frac{cTk}{2N}\right)^2}{R^2}$$

fractional terminals in the first bit-slot, and

$$\frac{\left(r+\frac{cTk}{2N}+icT\right)^2-\left(r-\frac{cTk}{2N}+icT\right)^2}{R^2}$$

in the i-th slot. There will be n_r such slots, where

$$n_r=\left[\frac{R-r}{cT}\right]$$

and $[\cdot]$ ≡ largest integer smaller than $(\cdot)$. Thus, the probability of interference to a terminal at range r is

$$P(I,r)=1-e^{-\sum_{i=0}^{n_r}\frac{2(r+icTk)cT}{NR^2}\lambda T_m}$$

The success probability for a terminal at r is:

$$P_s(r)=e^{-\lambda T_m f(r)}$$

and the throughput is

$$S = \int_0^R G e^{-G f(r)} p(r)\, dr$$

where

$$p(r) = \frac{2r}{R^2} \qquad 0 \le r \le R$$

$$= 0 \qquad \text{elsewhere}$$

and

$$f(r) = \frac{r^2}{R^2} + \sum_{i=0}^{n_r} \frac{2(r + icTk)}{NR^2} cT$$

If we put it all together, we have

$$S = \frac{2G}{R^2} \int_0^R e^{-\frac{G}{R^2}\left\{ r^2 + \frac{2(n_r+1)cTk}{N} r + \frac{n_r(n_r+1)(cT)^2 k}{2N} \right\}} r\, dr$$

This is not particularly nice; however, we can find a simple lower bound on S which will be acceptable for large N. First, if we replace n_r by $n = \max_r n_r = \frac{R}{cT}$ we have a lower bound given by

$$S \le \frac{2G}{R} e^{-\frac{Gk}{2N}\left(1 + \frac{cT}{R}\right)} \int_0^R e^{-G\left\{ \frac{r^2}{R^2} + \frac{2rk}{NR}\left(1 + \frac{cT}{R}\right) \right\}} \frac{r}{R}\, dr$$

If we define

$$a = \frac{k}{N}\left(1 + \frac{cT}{R}\right)$$

then by changing variables, completing the square, and glabfragging the jimwhockey we can integrate the bound to obtain:

$$S \le \frac{2G}{R} e^{-G\left(\frac{a}{2}-a^2\right)} \left\{ \sqrt{\frac{2}{G}} \left(e^{\frac{-Ga}{2}} - e^{\frac{-G(1+a^2)}{2}} \right) - a\,\mathrm{erfc}\left(\frac{1+a}{\sqrt{2}} \sqrt{G} \right) + a\,\mathrm{erfc}\left(a\sqrt{\frac{G}{2}} \right) \right\}$$

Since this expression provides insight of measure zero, curves have been computed for a typical system with k = 4, R = 10 miles, and T = 10μs (100 kBps bit rate).

These curves are shown in Fig. 4, together with curves for classic Aloha and slotted Aloha.

These curves show that very high throughput can be achieved only when channel traffic (G) is on the order of 3 or greater. In this situation, the aggregate message generation rate is such that more than one message is generated for every successful message. In this region the classic Aloha model is no longer a good model for real traffic, since the unperturbable sources will undoubtedly be perturbed if not frustrated.

RANGE ~ 10 M

BIT LENGTH ~ 10 μS

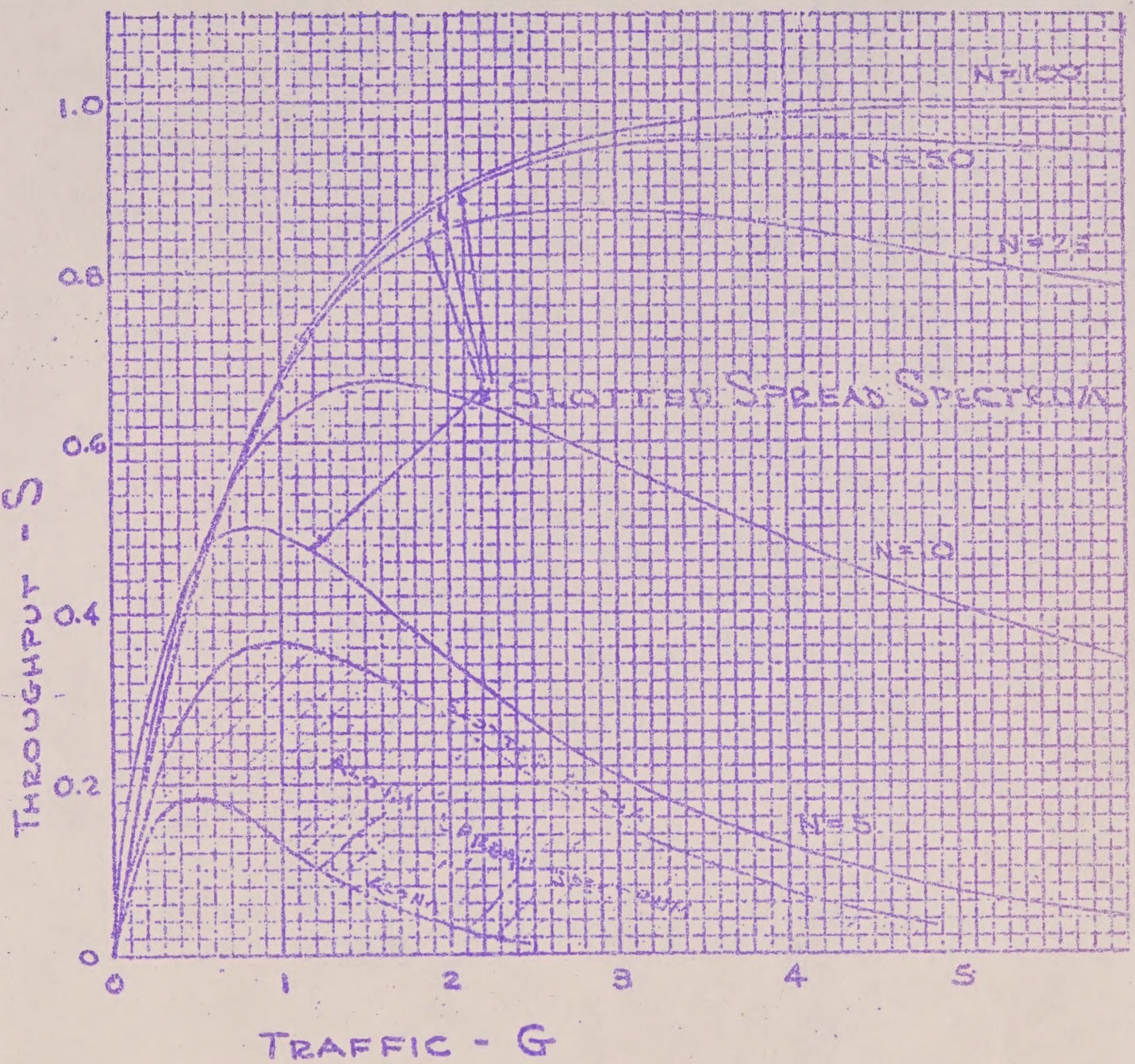


FIG 4 - THROUGHPUT VS TRAFFIC FOR SPREAD SPECTRUM PACKET RADIO

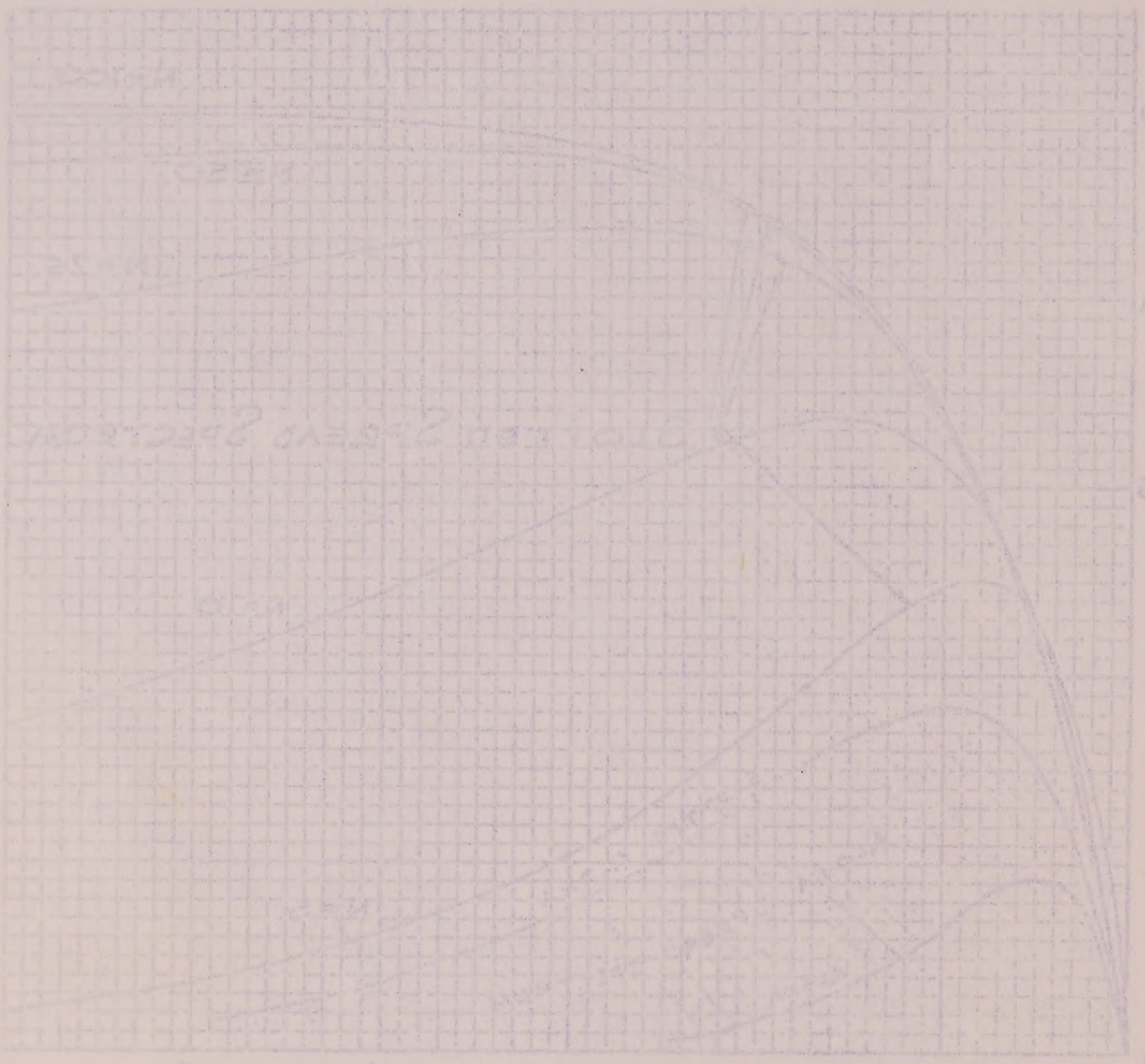


FIG 4- THROUGHPUT VS TRAFFIC FOR SPREAD SPECTRUM PACKET RADIO